Controllability Evaluation for Nonminimum Phase-Processes with Multiplicity

Annette Kuhlmann and I. David L. Bogle
Dept. of Chemical Engineering, Centre for Process Systems Engineering, University College London, London, WC1E 7JE, U.K.

Controllability describes the best achievable control quality independent of the controller design. If the achievable dynamic performance of a design is not good enough, then modifications to the plant design must be considered, such as changing inputs or outputs, adding or moving measurements, changing operating points, or even structural changes such as adding buffer tanks. This problem is addressed here by looking at the ability of a proposed design to move between operating points, that is, its switchability (White et al., 1996), which is an important aspect of the controllability. It is important when steps are intended regularly, for example, switches from day-time to night-time levels of operation, or varying customer demands, or if the process is to be run with an online optimizer that frequently provides new set points.

A way of obtaining insight into performance limitations is to consider an ideal controller, which is integral square error (ISE) optimal. This combines the various objectives on the quality and the speed of the response to a step input into a single measure. This has been considered for linear systems by Frank (1974), Holt and Morari (1985), and Morari and Zafiriou (1989). Also, Qiu & Davidson (1993) studied the "cheap regulator" problem and arrived at a result equivalent to the one by Holt and Morari, but derived in the time domain rather than in the frequency domain. It was shown that the ISE optimal closed-loop response can be determined via a factorization of the system $y = G(s)u$ (in the frequency domain) into a minimum and a nonminimum phase-part, such that $G(s) = G_+(s)\ G_-(s)$. The ISE optimal factorization is such that the nonminimum phase-factor $G_+(s)$ is an all-pass containing the system's right half plane (RHP) zeros. Frank describes this result as the unavoidable control error of nonminimum phase (NMP) systems. For systems with NMP behavior due to time delays, similar results are available in Frank (1974) and Morari and Zafiriou (1989).

General analysis is available for nonlinear systems from Van der Schaft (1996) who has shown that in general the solution to the ISE optimal factorization is analogous to the linear case, that is, an all-pass system and a minimum phase-system. Explicit solutions for control affine systems involves solving a Hamilton-Jacobi-Bellman equation. Relevant results on system inherent feedback limitations in nonlinear NMP systems have been presented by Seron et al. (1997). They study the cheap regulator problem and show that the output of an NMP system cannot be steered rapidly to zero, because it must first attend to the task of stabilizing the zero dynamics. It is shown that the limitations imposed by NMP behavior in nonlinear systems are analogous to the linear case, and cannot be overcome by a feedback controller.

An evaluation approach is required that identifies the process inherent limitation imposed by NMP behavior alone for nonlinear systems. The approach should be applicable to processes with input multiplicities, since nonlinear phenomena has been shown to be connected to NMP behavior under certain conditions (Sistu and Bequette, 1993; Jacobsen, 1994).

An analytical approach is shown for the evaluation of the NMP behavior of a nonlinear process. The applicability of the method is illustrated with a fermentation case study and then an approximate and more generally applicable approach to solve the problem is presented.

Controllability Evaluation for NMP Processes: Analytical Approach

In order to evaluate the controllability of nonlinear NMP processes the objective is to determine the best possible closed-loop response with respect to the ISE. This requires factorizing the system into minimum and NMP factors. Solutions are available for second-order nonlinear systems (Kravaris and Daoutidis, 1990) and for systems that can be transformed into natural coordinates (Wright and Kravaris, 1992). An NMP system implementing "perfect control" results in an unstable solution for the input u that produces the perfect control output. In the NMP case a stable solution to the problem is sought which gives the ISE optimal response.

The ISE optimization problem for a second-order process-model in normal form is a singular optimal control problem for which the solution can be obtained by following the procedure described in Bryson and Ho (1975). The singular control surface for this problem has two branches (singular arcs), which are the solution of the ISE optimization problem for the following two cases:

Correspondence concerning this article should be addressed to I. D. L. Bogle.

(1) When the system follows the singular Arc 1, it is perfectly controlled since the output is equal to the set point. The zero dynamics are stable only for minimum phase systems.

(2) When the system follows the singular Arc 2, it is not perfectly controlled and the closed-loop response can be determined. Following the singular Arc 2 will result in an internally stable closed-loop system if the system is NMP (Kravaris and Daoutidis, 1990). In terms of controllability this describes the best possible closed-loop performance of the process under idealized conditions, that is, the system inherent limitation to switchability.

The singular optimal control problem can be transformed into a nonsingular problem following the methods of Kelley (1965). Solving the ISE problem with this method leads to an expression for the minimum phase factor. For general NMP systems in natural coordinates, the calculation of the ISE optimal minimum phase factor requires the solution of an $(n-r)$th order integral of the Euler-Lagrange-Equation given in Wright and Kravaris (1992).

The construction of the nonminimum phase factor can be achieved by designing a perfect controller for y^* and determining the closed-loop response with the controller. Kravaris and Chung (1987) present a method for designing a globally linearizing controller (GLC) for y^* and the system in natural co-ordinates. Inserting this control law into the system in natural co-ordinates (and for $n=2$) yields the closed-loop system. This system represents the nonminimum phase-factor of the original process model in natural coordinates.

Case Study: Fermentation Process

The previously outlined method has been applied to the following model of a continuous fermenter

$$\dot{X} = \mu X - Xu$$
$$\dot{S} = \left(-\frac{\mu}{Y} - m\right)X - (S - S_f)u$$
$$y = X \tag{1}$$

The microbial growth is assumed to follow the Monod kinetics given by $\mu = \mu_{\max} S/(K_m + S)$, u is the manipulated input; in this case, the dilution rate and X and S are the concentrations of bacteria and substrate in the fermenter, respectively. The fermenter has input multiplicities. Table 1 shows the parameter values for the fermenter model.

The zero dynamics of the process are calculated to be

$$\dot{S} = \mu_{\max}(S_f - S)\frac{S}{K_m + S} + y\left(-m - \frac{\mu_{\max}}{Y}\frac{S}{K_m + S}\right) \tag{2}$$

Table 1. Parameter Values for Fermenter Model

Y	Yield Coefficient	0.4 g/g	m	Maintenance	0.01 h^{-1}
$\mu_{\max}$	Max. Specific Growth Rate	0.4 h^{-1}	K_m	Monod Const	0.05 g/L

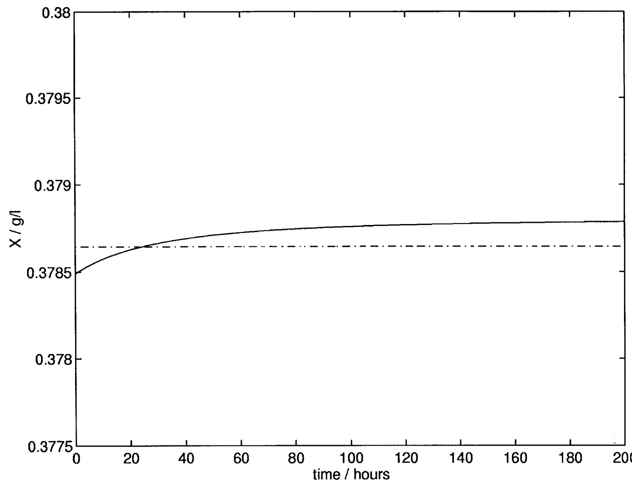

Figure 1. ISE optimal step response for an input step in D from 0.11 to 0.12 L/h.

Dash dotted line indicates the original steady-state.

The dilution rate at which the zero of the linearized transfer function crosses the imaginary axis is $u = 0.1213\ h^{-1}$, which is also the input at which the gain is zero. The steady state ($X = 0.379$ g/L, $S = 0.019$ g/L, $D = 0.11\ h^{-1}$) is in the NMP region.

Applying the transformation $\xi_1 = S_f X/(S_f - S)$ and $\xi_2 = X$ on the model leads to the normal form with

$$F_1 = \mu\xi_1 + \left(-\frac{\mu}{Y} - m\right)\frac{\xi_1^2}{S_f}$$
$$F_2 = \mu\xi_2$$
$$G_2 = -\xi_2 \tag{3}$$

Since F_1 is independent of ξ_2, the model needs to be transformed into natural coordinates. The minimum phase factor has, as expected, no inverse response and yields the same steady-state value as the original system. To demonstrate the effect of the nonminimum phase factor, the ISE optimal closed-loop response of the system to an input step $D = 0.11\ h^{-1}$ to $D = 0.12\ h^{-1}$ is shown in Figure 1. It was determined via simulation of the closed-loop system and appending to this system the ISE equation. The original steady state in this plot is indicated by the dash-dotted line. The figure shows that the optimal closed-loop response displays all-pass type behavior as in the linear case, that is, a sharp inverse response.

This response is the best that can be achieved for this set-point step change employing a causal controller in an idealized case. No input constraints, disturbances, or modeling errors have been considered. It is the limitation imposed by the presence of the unstable zero dynamics only.

The output step corresponding to the input step from 0.11 h^{-1} to 0.12 h^{-1} is $\Delta y_{\text{ref}} = 2.0 \times 10^{-4}$ g/L. This indicates a best possible ISE_{lin} in the linearized case of 1.41×10^{-6} at the steady state considered. The nonlinear analysis shows that this step can be achieved with an ISE of 1.89×10^{-6} in the best case. This is 134% of the value predicted with the linear analysis. A symmetric negative step yields a best possible ISE of 1.23×10^{-6}, which corresponds to only 87% of the linearized value.

This result can be explained by considering the RHP zero position of the process corresponding to the steady states. From the steady state considered ($D=0.11\ h^{-1}$), the process zero moves towards the imaginary axis for the positive step (to $D=1.2\ h^{-1}$). For the linear analysis, this corresponds to stronger limitations on controllability at the new steady state. This is reflected in the nonlinear result.

Conclusions from the Analytical Method

If the process is moved towards a stronger NMP steady-state, the linear analysis underestimates the controllability limitation imposed by NMP behavior. In this way the nonlinearity of the zero dynamics of the NMP process is reflected in the limitation these zero dynamics impose on the controllability.

The analysis answers the question whether or not the achievable performance is sufficient for the demands imposed on the process. If the answer is negative, no causal controller exists that will be able to improve performance any further, that is, the limitations are system inherent. Such a design should be discarded or changed, in order to improve the controllability. If the answer to the question is positive, it is, of course, still not certain that a controller actually exists that can provide the required performance.

There are some restrictions to the method. The approach can only be applied to second-order systems in normal form and to systems of relative order order $r=n-1$ that can be transformed into natural co-ordinates. The analysis still has to be done for a range of set point changes, and no direct conclusion can be drawn as to which design parameter should be changed in order to improve the situation.

One of the limitations of the analytical evaluation method concerns the input multiplicity phenomenon. Processes with input multiplicities have, under some assumptions, operating regions with NMP behavior (Sistu and Bequette, 1993; Jacobsen, 1994). Hence, an evaluation method cannot be applied in both regions because the two regions have different solutions to the ISE problem. This means that steps that cross from one region to another, or, in other words, cross the maximum in the steady-state curve, cannot be evaluated with this method.

A different method using optimization to evaluate the controllability of NMP nonlinear systems that overcomes most of these problems is discussed next.

Controllability Evaluation for NMP System: An Approximation Approach

The optimization problem for NMP systems that determines the ISE of a step from one steady state to another in order to evaluate controllability of a given design cannot be solved numerically due to two major problems: an infinite time horizon is not implementable and the solution ISE = 0, that is, the "true" minimum of the problem is unstable for NMP systems.

To overcome this problem, an endpoint constraint that ensures that the process is at the new steady state and, therefore, that a stable solution has been found has to be included in the formulation. An endpoint constraint on the output is not sufficient due to the existence of an unstable solution with constant output in an NMP system. Also, the frequent occurrence of input multiplicity in NMP processes makes it necessary to ensure that the desired input-value is reached at the new steady state.

The idea used here comes from the quasi-infinite horizon approach (Chen and Allgöwer, 1998) for nonlinear MPC. The time horizon T is split into two parts

$$\min_u \text{ISE} = \frac{1}{2}\int_0^{T_1}[y_{\text{ref}} - y(t)]^2 dt + \frac{1}{2}\int_{T_1}^{\infty}[y_{\text{ref}} - y(t)]^2 dt \tag{4}$$

subject to the model and given initial conditions.

The second-part of the integral still cannot be solved. However, the problem can be formulated differently using a finite-time horizon. A time horizon T_2 of sufficient length to ensure the system reaches steady state is necessary. T_2 has to be chosen long enough to ensure feasibility, while being short enough for the problem to be solved in reasonable time. The numerical necessity to discretize the time horizon implies that the solution becomes inaccurate if the horizon is chosen to be too long.

The second integral is implemented with only one step of undefined length, determined by the endpoint constraint in the state derivatives. The problem can be solved without knowing the necessary horizon time in advance. The first integral of the objective function serves to find the minimum ISE solution. This means that T_1 should be as long as is practical for implementation and with as many time steps as possible, such that the solution is close to the continuous solution.

If the process has input multiplicities an end-point constraint on the input has to be included. A straightforward way is to analyze the steady-state relation and find the maximum in the steady-state curve. Then, if a step inside the NMP region is to be investigated, the final input-value has to be constrained to be inside this region after the step.

The second integral of Eq. 4, if implemented with a one-time-step only, equals the open-loop response of the system from the state the system reached during the time T_1; the ISE of this open-loop response can in the best case be equal to the ISE optimal solution, but is likely to be nonoptimal. In that sense the solution yields an upperbound on the best achievable ISE. Having found an upperbound is a little unfortunate, since what is sought is the best achievable ISE. However, if the objective is applied as a nonlinear MPC to the process, this upperbound turns into the best achievable performance with this controller. If, however, the determined achievable control performance is not sufficient for the demands imposed on the process operation, this could be due to the approximation used in the method. In this case further studies are necessary.

Comparison Between Analytical and Approximate Solutions

Where it can be shown analytically that two solutions exist to the problem, one stable and one unstable, the globally op-

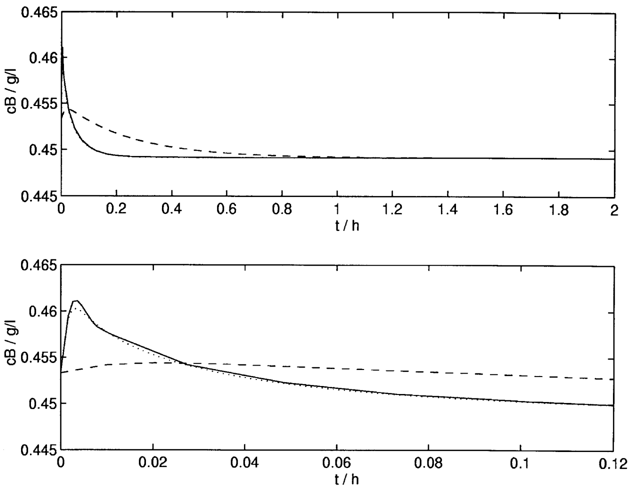

Figure 2. Optimal closed-loop responses for Van de Vusse reactor: in two time scales.

Analytical solution (dotted), approximate solution (solid), open-loop response (dashed).

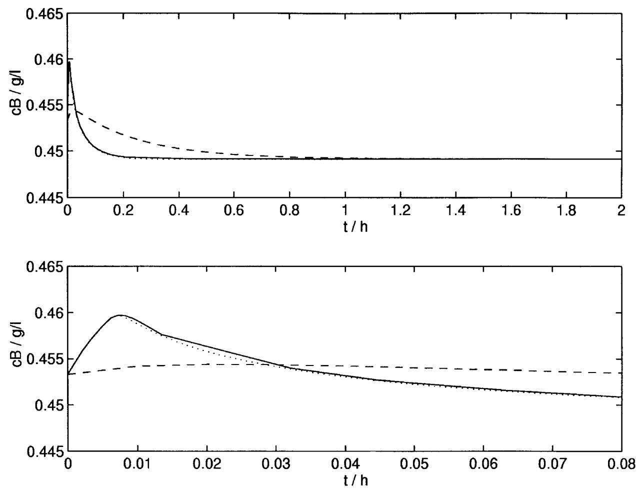

Figure 3. Optimal closed-loop responses for ISE optimized solution for Van de Vusse reactor with input constraints: in two time scales.

Analytical solution (dotted), approximate solution (solid), open-loop response (dashed).

timal solution of the ISE optimal factorization problem is found. For the Van de Vusse example (Kuhlmann and Bogle, 1997), the approximate formulation gives the globally optimal solution. The final control interval (from T_1 to T_2) has a long minimum length, which is necessary in order to avoid the unstable solution.

The stable solution with optimal ISE for a step in the process input is shown in Figure 2, where the upper plot shows the entire time interval necessary to reach the new steady state and the lower plot shows a detail at the beginning of the step. The solid line is the solution determined numerically, while the dotted line is the analytical solution. The dashed line shows the corresponding open-loop response of the process output. For the nonlinear analytical method, input constraints do not present a problem as the solution is still optimal when input constraints are imposed (Bryson and Ho, 1975). The solution is at the bounds as long as necessary and then travels along the singular arcs.

The optimal closed-loop responses with the input restricted to positive values determined both analytically and numerically for the previous case study are shown in Figure 3. The values of the ISE found with the two different methods are given in Table 2. For this case, it is known that the analytical solution is globally optimal. The values of the ISE found with the two methods match very closely in both the constrained and the unconstrained cases. Although this is not necessarily true, in general, the method yields a valid approximation for the process shown and is thus believed to be applicable for other processes too.

Fermentation Case Study: Crossing the Maximum in the Steady-State Curve

Particular problems arise when operating a process across the region where both minimum phase and NMP behavior occur. For the fermentation process, if the bacteria are now assumed to need no substrate for their maintenance, but will die at a certain rate, the model is subject to input-multiplicities and has NMP behavior in part of the operating range. Competing physical effects on the bacteria concentration, the output of the process, cause the multiplicity.

The model equations are as follows

$$\dot{X} = (\mu - K_d)X - Xu$$

$$\dot{S} = -\frac{\mu}{Y}X + (S_f - S)u$$

$$y = X \tag{5}$$

The growth-rate is assumed to follow the Monod kinetics as before. The model parameters are the same as previously described and the decay rate is $K_d = 0.04\ \text{h}^{-1}$.

The fermenter is assumed to operate at a steady rate with a dilution rate of 0.15 h^{-1}, where the fermenter is NMP. At certain regular periods in the production cycle, the fermenter is operated at higher dilution rates due to an increased product demand even though the risk of washout is greatly increased on the minimum phase-side of the steady-state curve.

The result of the analysis is shown in Figure 4. Each circle corresponds to the solution of one dynamic optimization problem. It shows the smallest achievable ISE for a step from the steady state at 0.15 h^{-1}. The best possible ISE increases for steps towards the maximum in the steady-state curve, and decreases for further increasing steps across the maximum. The ISE reaches zero for a step to the input multiplicity steady state with exactly the same output value. This is as expected, since switching between the two inputs cannot be

Table 2. ISE of Closed-Loop Responses

	Analytical Solution	Approximate Solution
Unconstrained problem	2.2640×10^{-6}	2.3280×10^{-6}
Constrained problem	2.3392×10^{-6}	2.3616×10^{-6}

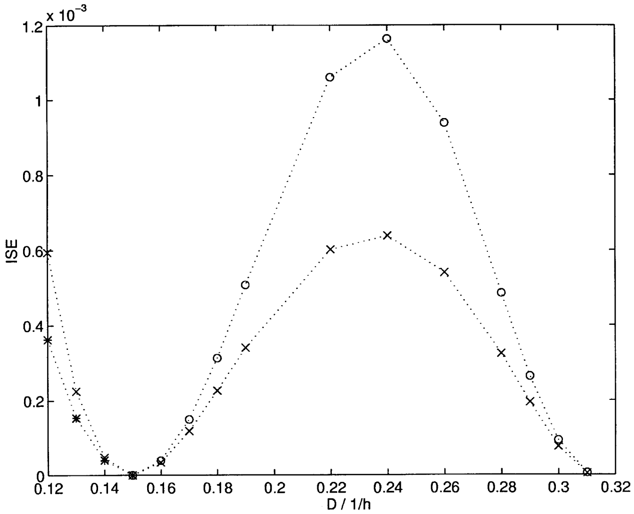

Figure 4. Linear (crosses) and nonlinear (circles for steps toward and stars for steps away from the maximum in the steady-state curve) ISE optimal solutions for steps from steady state at 0.15 L/L.

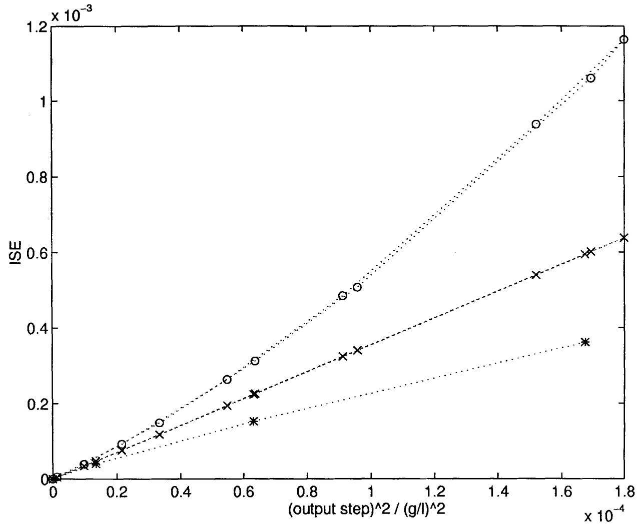

Figure 5. Linear (crosses) and nonlinear (circles for steps toward and stars for steps away from the maximum in the steady state curve) ISE optimal solutions for steps from steady state at 0.15 L/L.

seen at the output of such a process, that is, it is realizable with zero ISE (Abu el Ata-Doss et al., 1992). The stars indicate the ISE obtained for negative steps away from the maximum in the steady-state curve. The achievable ISE for decreasing inputs has less effect on the controllability of the process. The crosses in the plot show the values obtained with a linear analysis giving an indication of the degree of nonlinearity in the region.

A different method of presenting the result is employed in Figure 5. The achievable ISE (again each circle or star for one dynamic optimization problem solved) is plotted against the squared output step-size. This is done because in this format the linear measure is a straight line shown by the crosses.

Figure 5 again shows that the best achievable ISE for the nonlinear process for steps towards the maximum is bigger than the value expected from a linear analysis. If the switch requires the process to move across the maximum, the ISE increases along the same line as before. This is due to the fact that steps between two steady states with the same output value can be achieved with zero ISE.

Conclusions

Two approaches to determine the process inherent limitation on switchability imposed by unstable zero dynamics have been shown. The first solves the problem analytically and results in an expression for the NMP factor, which dictates the system inherent limitation. The method is restricted to two classes of rather small systems and also cannot be applied to models that have both NMP and MP regions. The second which overcomes this restriction uses a method numerically optimizing the dynamic model in order to find the ISE optimal closed-loop response. Due to the fact that the process is NMP, some precautions were taken when the problem was formulated in order to avoid finding the unstable inverse. The method was also used to study the system's inherent limitations for steps over the maximum in a process with input multiplicities. The nonlinearities do have an effect, but the extent will, of course, vary from problem to problem. The problems are still small, and there is some way to go before we have a method for problems of realistic size.

Acknowledgments

The authors would like to thank the EPSRC of the U.K. for financial support.

Literature Cited

Abu el Ata-Doss, S., A. Coic, and M. Fliess, "Non Linear Predictive Control by Inversion: Discontinuities for Critical Behavior," *Int. J. Control*, **55**, 6, 1521 (1992).

Bryson, Jr., A. E., and Yu-Chi Ho, *Applied Optimal Control*, Hemisphere Pub. Coop. Washington, DC (1975).

Chen, H., and F. Allgöwer, "A Quasi-Infinite Horizon Nonlinear Model Predictive Control Scheme with Guaranteed Stability," *Automatic*, **34**, 1205 (1998).

Frank, P. M., *Entwurf von Regelkreisen mit Vorgeschriebenem Verhalten*, G. Braun, Karlsruhe, Germany (1974).

Holt, B. R., and M. Morari, "Design of Resilient Processing Plants–VI. The Effect of Right-Half-Plane Zeros on Dynamic Resilience," *Chem. Eng. Sci.*, **40**, 59 (1985).

Jacobsen, E. W., "Dynamics of Systems with Steady State Input Multiplicity," AIChE Meeting, San Francisco (1994).

Kelley, H. J., "A Transformation Approach to Singular Subarcs in Optimal Trajectory and Control Problems," *Siam J. Control*, **2**, 2, 234 (1965).

Kravaris, C., and C.-B. Chung, "Nonlinear State Feedback Synthesis by Global Input/Output Linearisation," *AIChE J.*, **33**(4), 592 (1987).

Kravaris, C., and P. Daoutidis, "Nonlinear State Feedback Control for Second Order Nonminimum Phase Nonlinear Systems," *Comp. Chem. Eng.*, **14**, 439 (1990).

Kuhlmann, A., and D. Bogle, "Study on Nonminimum Phase Behaviour and Optimal Operation," *Comp. Chem. Eng.*, **21**, S397 (1997).

Morari, M., and E. Zafiriou, *Robust Process Control*, Prentice Hall, Englewood Cliffs, NJ (1989).
Qui, L., and E. J. Davidson, "Performance Limitations of Non-Minimum Phase Systems in the Servomechanism Problem," *Automatica*, **29**, 2, 337 (1993).
Seron, M. M., J. H. Braslavsky, P. V. Kokotovic, and D. Q. Mayne, "Feedback Limitations in Nonlinear Systems: From Bode Integrals to Cheap Control," *Proc. IEEE Conf. on Decision and Control IEEE*, New York, **3**, 2067 (1997).
Sistu, P. B., and B. W. Bequette, "Model Predictive Control of Process with Input Multiplicities," AIChE Meeting, St. Louis, MO (1993).
Van der Schaft, A., "L_2-Grain and Passivity Techniques in Nonlinear Control," *Lecture Notes in Control and Information Sciences 218*, Springer-Verlag, Berlin (1996).
White, V., J. D. Perkins, and D. M. Espie, "Switchability Analysis," *Comp. Chem. Eng.*, **20**, 469 (1996).
Wright, R. A., and C. Kravaris, "Nonminimum-Phase Compensation for Nonlinear Processes," *AIChE J.*, **38**, 1, 26 (1992).

Manuscript received June 2, 1998, and revision received Feb. 28, 2001.